## Selection maintaining protein stability at equilibrium



Sanzo Miyazawa
sanzo.miyazawa@gmail.com
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The common understanding of protein evolution:

- Most amino acid substitutions observed in homologous proteins were selectively neutral and fixed by random drift.
- A proportion of neutral mutations that depends on the strength of structural and functional constraints primarily determines evolutionary rate.

Recently a question has been raised on the common view of protein evolution.

- There are a diversity of protein evolutionary rates among genes.
- Protein evolutionary rate is correlated with gene expression level; highly expressed genes evolve slowly.
- Fitness costs due to misfolded proteins are a determinant of evolutionary rate and selection originating in protein stability is a driving force of protein evolution.

Here we examine protein evolution under the selection maintaining protein stability.

## Functional loss and toxicity due to misfolded proteins

Functional loss and toxicity caused by misfolded proteins (Drummond et al., 2005):

- Misfolding reduces the concentration of functional proteins.
- Misfolding wastes cellular time and energy on production of useless proteins.
- Misfolded proteins form insoluble aggregates.

Fitness cost due to misfolded proteins is larger for highly expressed genes than for less expressed ones (Geiler-Samerotte et al., 2011).

## A generic form of fitness costs due to protein misfolding

- Malthusian fitness for protein dispensability (Drummond et al., 2008):

$$
\begin{equation*}
m_{\text {dispensability }} \equiv-\sum_{i} \gamma_{i}\left(1-f_{i}^{\text {native }}\right) \tag{1}
\end{equation*}
$$

- Malthusian fitness for toxicity of misfolded proteins (Drummond et al., 2008):

$$
\begin{equation*}
m_{\text {misfolds }}=-c \sum_{i} A_{i} \frac{1-f_{i}^{\text {native }}}{f_{i}^{\text {native }}} \tag{2}
\end{equation*}
$$

- Selection to maintain protein stability (Dasmeh et al., 2014):

$$
\begin{equation*}
m=\log f^{\text {native }} \tag{3}
\end{equation*}
$$

The proportion of native conformations, $f^{\text {native }}$, in a two state transition:

$$
\begin{equation*}
f^{\text {native }}=\frac{e^{-\beta \Delta G}}{1+e^{-\beta \Delta G}} \tag{4}
\end{equation*}
$$

where $\Delta G$ is the folding free energy of protein.
Because $\exp \beta \Delta G \ll 1$ for typical proteins, all these formulas of Malthusian fitness for misfolded proteins are reduced to

$$
\begin{equation*}
m \equiv-\sum_{i} \kappa_{i} e^{\beta \Delta G_{i}} \quad \text { with } \kappa_{i} \geq 0 \tag{5}
\end{equation*}
$$

## 3. Methods

## The evolution of a single coding gene in a monoclonal approximation

Here, we consider the evolution of a single protein-coding gene in which the selective advantage of mutant proteins in Malthusian parameters is assumed to be

$$
\begin{align*}
s & \equiv m^{\text {mutant }}-m^{\text {wildtype }}  \tag{6}\\
4 N_{e} s & =4 N_{e} \kappa e^{\beta \Delta G}\left(1-e^{\beta \Delta \Delta G}\right) \quad \text { with } \kappa \geq 0 \tag{7}
\end{align*}
$$

If the fitness costs of functional loss and toxicity due to misfolded proteins are both taken into account and assumed to be additive in the Malthusian fitness scale, $\kappa$ will be defined as

$$
\begin{equation*}
\kappa=c A+\gamma \tag{8}
\end{equation*}
$$

| $c$ | $\sim 10^{-4}$ | fitness cost per misfolded protein |
| :--- | :--- | :--- |
| $A$ | $10<A<10^{6}$ | cellular abundance of protein |
| $\gamma$ | $0 \leq \gamma \leq 10$ | protein indispensability |
| $N_{e}$ |  | effective population size |
|  | $\sim 10^{4}$ to $10^{5}$ | for vertebrates |
|  | $\sim 10^{5}$ to $10^{6}$ | for invertebrates |
|  | $\sim 10^{7}$ to $10^{8}$ | for unicellular eukaryotes |
|  | $>10^{8}$ | for prokaryotes |

## Stability changes, $\Delta \Delta G$, due to single amino acid substitutions

PDF approximated with a weighted sum of two Gaussian functions (Tokuriki et al., 2007):

$$
\begin{equation*}
p(\Delta \Delta G)=\theta \mathcal{N}\left(\mu_{s}, \sigma_{s}\right)+(1-\theta) \mathcal{N}\left(\mu_{c}, \sigma_{c}\right) \tag{9}
\end{equation*}
$$

$$
\begin{align*}
\text { For surface residues : } & \mu_{s}=-0.14 \Delta G-0.17, \quad \sigma_{s}=0.90  \tag{10}\\
\text { For core residues : } & \mu_{c}=-0.14 \Delta G+1.23, \quad \sigma_{c}=1.93 \tag{11}
\end{align*}
$$

The dependences of the means, $\mu_{c}$ and $\mu_{s}$, on $\Delta G$ are estimated from the regression line of observed values of $\Delta \Delta G$ of mutant proteins on $\Delta G$ of the wild-type protein.


Solid: regression; Broken: $\mu_{c}$, Dotted: $\mu_{c} \pm \sigma_{c}$; Broken: $\mu_{s}$, Dotted: $\mu_{s} \pm \sigma_{s}$

## PDFs of $4 N_{e} s$ and $K_{a} / K_{s}$

Instead of pursueing computer simulations of gene populations, we calculate the probability density functions (PDF) of characteristic quantities such as selective advantage, fixation probabilty, and $K_{a} / K_{s}$, and examine the protein evolution of the gene in a monoclonal approximation.

Fixation probability:

$$
\begin{equation*}
u\left(4 N_{e} s\right)=\frac{1-e^{-4 N_{e} s q}}{1-e^{-4 N_{e} s}} \tag{12}
\end{equation*}
$$

where $q=1 /(2 N)$ for a mutant gene, and $N$ is a population size. Population size is taken to be $N=10^{6}$.

The ratio of nonsynonymous $\left(K_{a}\right)$ to synonymous substitution rate per site $\left(K_{s}\right)$ :

$$
\begin{equation*}
\frac{K_{a}}{K_{s}}=\frac{u\left(4 N_{e} s\right)}{u(0)}=\frac{u\left(4 N_{e} s\right)}{q} \quad \text { with } q=\frac{1}{2 N} \tag{13}
\end{equation*}
$$

PDF of $4 N_{e} s: p\left(4 N_{e} s\right)=-p(\Delta \Delta G) \frac{d \Delta \Delta G}{d 4 N_{e} s}$
PDF of $K_{a} / K_{s}: p\left(K_{a} / K_{s}\right)=p\left(4 N_{e} s\right) \frac{d 4 N_{e} s}{d u} \frac{d u}{d\left(K_{a} / K_{s}\right)}$

## PDFs of $\Delta \Delta G, 4 N_{e} s$, and $K_{a} / K_{s}$ in fixed mutants

PDF of $\Delta \Delta G$ in fixed mutants:

$$
\begin{align*}
p\left(\Delta \Delta G_{\mathrm{fixed}}\right) & \equiv p(\Delta \Delta G) \frac{u\left(4 N_{e} s\right)}{\langle u\rangle}  \tag{14}\\
\langle u\rangle & \equiv \int_{-\infty}^{\infty} u\left(4 N_{e} s\right) p(\Delta \Delta G) d \Delta \Delta G \tag{15}
\end{align*}
$$

PDF of $4 N_{e} s$ in fixed mutants:

$$
\begin{equation*}
p\left(4 N_{e} s_{\text {fixed }}\right)=-p\left(\Delta \Delta G_{\mathrm{fixed}}\right) \frac{d \Delta \Delta G}{d 4 N_{e} s} \tag{16}
\end{equation*}
$$

PDF of $K_{a} / K_{s}$ in fixed mutants:

$$
\begin{equation*}
p\left(\left(K_{a} / K_{s}\right)_{\text {fixed }}\right)=p\left(4 N_{e} s_{\text {fixed }}\right) \frac{d 4 N_{e} s}{d u} \frac{d u}{d\left(K_{a} / K_{s}\right)} \tag{17}
\end{equation*}
$$

## 4. Results

## PDFs of stability change, $\Delta \Delta G$,

due to single amino acid substitutions at equilibrium stability, $\Delta G=\Delta G_{e}$, where $\langle\Delta \Delta G\rangle_{\text {fixed }}=0$.
in all mutants


## Equilibrium stability, $\Delta G_{e}$

The average, $\langle\Delta \Delta G\rangle_{\text {fixed }}$, of stability changes over fixed mutants versus protein stability, $\Delta G$, of the wild type.

$\Delta G_{e}$ is the stable equilibrium point for $\Delta G$, where $\langle\Delta \Delta G\rangle_{\mathrm{fixed}}=0$.

Dependence of equilibrium stability, $\Delta G_{e}$, on parameters, $4 N_{e} \kappa$ and $\theta$.



- The value of $\beta \Delta G_{e}+\log 4 N_{e} \kappa$ is the upper bound of $\log 4 N_{e} s$, and would be constant if the mean of $\Delta \Delta G$ in all arising mutants did not depend on $\Delta G$.
- $\Delta G_{e}$ decreases as $\log 4 N_{e} \kappa$, effective population size or protein abundance/indispensability, increases.


## Distribution of folding free energies of monomeric protein families



- The observed range of $\Delta G$ shown above is consistent with that range, -2 to $-12.5 \mathrm{kcal} / \mathrm{mol}$, expected from the present model.

- Protein abundance/indispensability and effective population size, $4 N_{e} \kappa$, more decrease evolutionary rate for less-constrained proteins.
- Structural constraint, $1-\theta$, more decreases evolutionary rate for less-abundant, less-essential proteins.
- $\left\langle K_{a} / K_{s}\right\rangle<1$ over a whole range of the parameters.


# PDFs of $K_{a} / K_{s}$ at equilibrium of protein stability, <br> <br> $\Delta G=\Delta G_{e}$, where $\langle\Delta \Delta G\rangle_{\text {ixixed }}=0$. 

 <br> <br> $\Delta G=\Delta G_{e}$, where $\langle\Delta \Delta G\rangle_{\text {ixixed }}=0$.}
in all mutants

in fixed mutants

## Probability of each selection category

in fixed mutants at equilibrium of protein stability, $\Delta G=\Delta G_{e}$.

- Nearly neutral selection is predominant only for low-abundant, non-essential proteins.
- Positive selection is significant for the other proteins.
nearly neutral selection, $P\left(0.95<\left(K_{a} / K_{s}\right)_{\text {fixed }}<1.05\right) \quad$ positive selection, $P\left(1.05<\left(K_{a} / K_{s}\right)_{\text {fixed }}\right)$




slightly negative selection, $P\left(0.5<\left(K_{a} / K_{s}\right)_{f i x e d}<0.95\right)$
negative selection, $P\left(\left(K_{a} / K_{s}\right)_{f i x e d}<0.5\right)$
- Slightly negative selection is always significant.


## Dependence of each selection on $4 N_{e} \kappa$ and $\triangle G$

in fixed mutants; shown within $2 \cdot \Delta \Delta G_{\text {fixed }}^{\text {sd }}$ around $\Delta G=\Delta G_{e}$ indicated by a blue line.

- Positive selection is predominant in $\Delta G>\Delta G_{e}$.
- Nearly neutral and slightly negative selections are predominant in $\Delta G<\Delta G_{e}$.
nearly neutral selection
positive selection



## Dependence of each selection on structural constraint $(\theta)$ and $\triangle G$

in fixed mutants; shown within $2 \cdot \Delta \Delta G_{\text {fixed }}^{\text {sd }}$ around $\Delta G=\Delta G_{e}$ indicated by a blue line.

- Positive selection is predominant in $\Delta G>\Delta G_{e}$.
- Nearly neutral and slightly negative selections are predominant in $\Delta G<\Delta G_{e}$.
nearly neutral selection
positive selection



## Dependence of the average of $K_{a} / K_{s}$ on $\Delta G$ of the wild type;

shown within $2 \cdot \Delta \Delta G_{\text {fixed }}^{\text {sd }}$ around $\Delta G=\Delta G_{e}$ indicated by a blue line.

- $\left\langle K_{\mathrm{a}} / K_{s}\right\rangle<1$ but $\left\langle K_{\mathrm{a}} / K_{s}\right\rangle_{\text {fixed }}>1$ in $\Delta G>\Delta G_{e}$.
- $\left\langle K_{a} / K_{s}\right\rangle_{\text {fixed }} \sim 1$ in $\Delta G<\Delta G_{e}$, in which nearly neutral selection is predominant.



## Dependence of equilibrium stability, $\Delta G_{e}$, on growth temperature $T$

- Protein stability $\left(-\Delta G_{e} / k T\right)$ is predicted to decrease as growth temperature increases.

- Evolutionary rate may be predicted from $\theta$ and $\Delta G_{e}$ rather than $4 N_{e} \kappa$.

- The range, -2 to $-12.5 \mathrm{kcal} / \mathrm{mol}$, of equilibrium values, $\Delta G_{e}$, of protein stability calculated with the present fitness model is consistent with the distribution of experimental values.
- Contrary to the neutral theory, nearly neutral selection is predominant only in low-abundant, non-essential proteins of $\log 4 N_{e} \kappa<2$ or $\Delta G_{e}>-2.5 \mathrm{kcal} / \mathrm{mol}$. In the other proteins, positive selection on stabilizing mutations is significant to maintain protein stability at equilibrium as well as random drift on slightly negative mutants. However, $\left\langle K_{a} / K_{s}\right\rangle$ and even $\left\langle K_{a} / K_{s}\right\rangle_{\text {fixed }}$ at $\Delta G=\Delta G_{e}$ are less than 1.
- Protein abundance/indispensability ( $\kappa$ ) and effective population size $\left(N_{e}\right)$ more affect evolutionary rate for less constrained proteins, and structural constraint $(1-\theta)$ for less abundant, less essential proteins.
- Protein indispensability must negatively correlate with evolutionary rate like protein abundance, but the correlation between them may be hidden by the variation of protein abundance and detected only in low-abundant proteins.
- The present model indicates that protein stability $\left(-\beta \Delta G_{e}\right)$ and $\left\langle K_{a} / K_{s}\right\rangle$ decrease as growth temperature increases.

